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Parameter inference with estimated covariance matrices

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Of importance for:

Euclid, DES, CFHTLenS, KiDS, BOSS, eBOSS,

all analyses that use the

covariance matrix estimator

$$S = \frac{1}{N-1}\sum_{i=1}^{N}(\vec{x}_i - \bar{\vec{x}})(\vec{x}_i - \bar{\vec{x}})^T$$

Parameter inference from Gaussian data:

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$$\mathcal{G}(\vec{x}|\vec{\mu}(\vec{p}),C) = \frac{1}{\sqrt{|2\pi C|}} \exp\left(-\frac{1}{2}(\vec{x}-\vec{\mu})^T C^{-1}(\vec{x}-\vec{\mu})\right)$$

→ C is not a random object – the estimator S is

→ $C^{-1} = \alpha\langle S^{-1}\rangle$ where $\alpha = \frac{N-p-2}{N-1}$

→ Hartlap et al. (2007): Gaussian with $C^{-1} \rightarrow \alpha S^{-1}$

But: Removing <> means ignoring the randomness of S!

Hartlap et al.'s method from (2007):

$$C^{-1} \to \alpha S^{-1}$$

It addresses an important problem
– but it overcompensates in the solution.

- It leads to biased confidence contours
- It finds too often too good a fit
- It rejects null-hypotheses too early
- For weak lensing (DES):
  - It increases the error bars by too much (about 10%)
  - It forces you to use 75% more simulations than actually necessary

The Bayesian way to replace C → S :

$$L(\vec{x}|\vec{\mu}, S) = \int \mathcal{G}(\vec{x}|\vec{\mu}, C)\underline{\underline{\mathcal{P}(C|S, N)}}\mathrm{d}C$$ ← matrix integration

matrix-valued prob. dens. func.

- $\mathcal{P}(S|C) = \text{Wishart distribution}$
- $\pi(C) \propto |C|^{-(p+1)/2}$

$$\rightarrow \mathcal{T}(\vec{x}|\vec{\mu}, S) \propto \frac{|S|^{-1/2}}{\left[1 + \frac{(\vec{x}-\vec{\mu})^T S^{-1} (\vec{x}-\vec{\mu})}{N-1}\right]^{\frac{N}{2}}}$$

Eq. (12) in arXiv 1511:05969 (changes 3 lines of codes)

(Un)biased confidence contours

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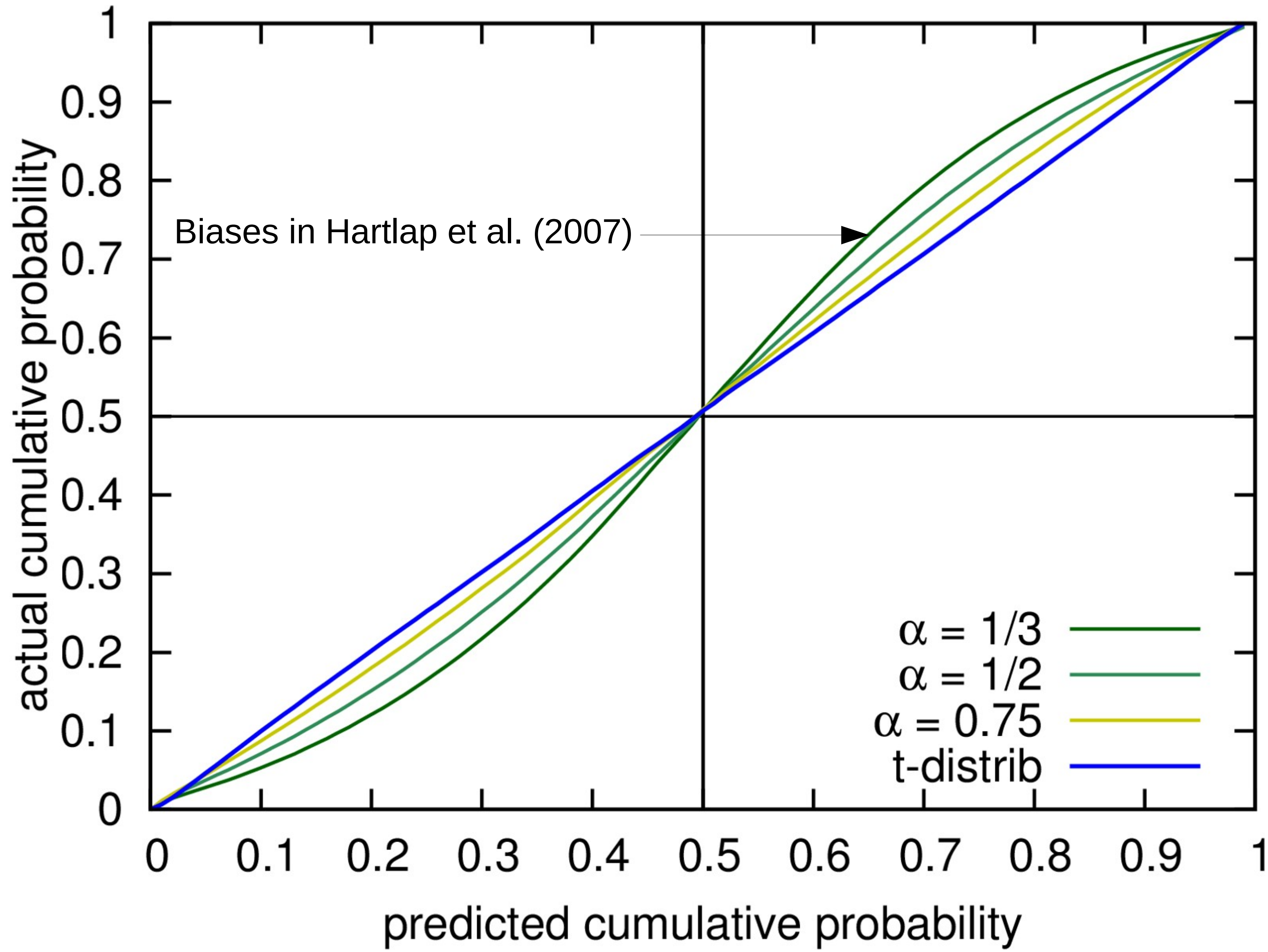
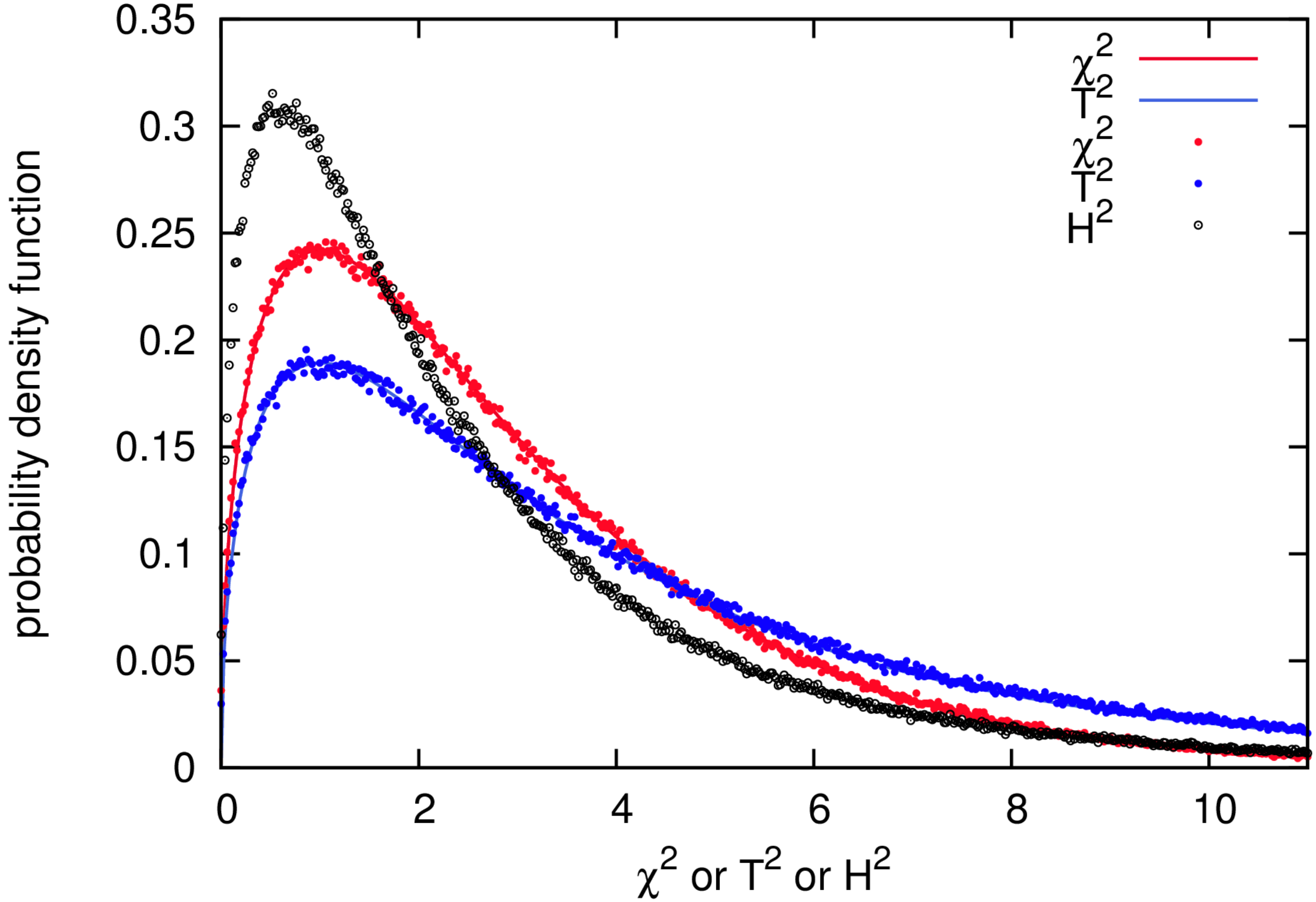


Goodness of fit and statistical scatter

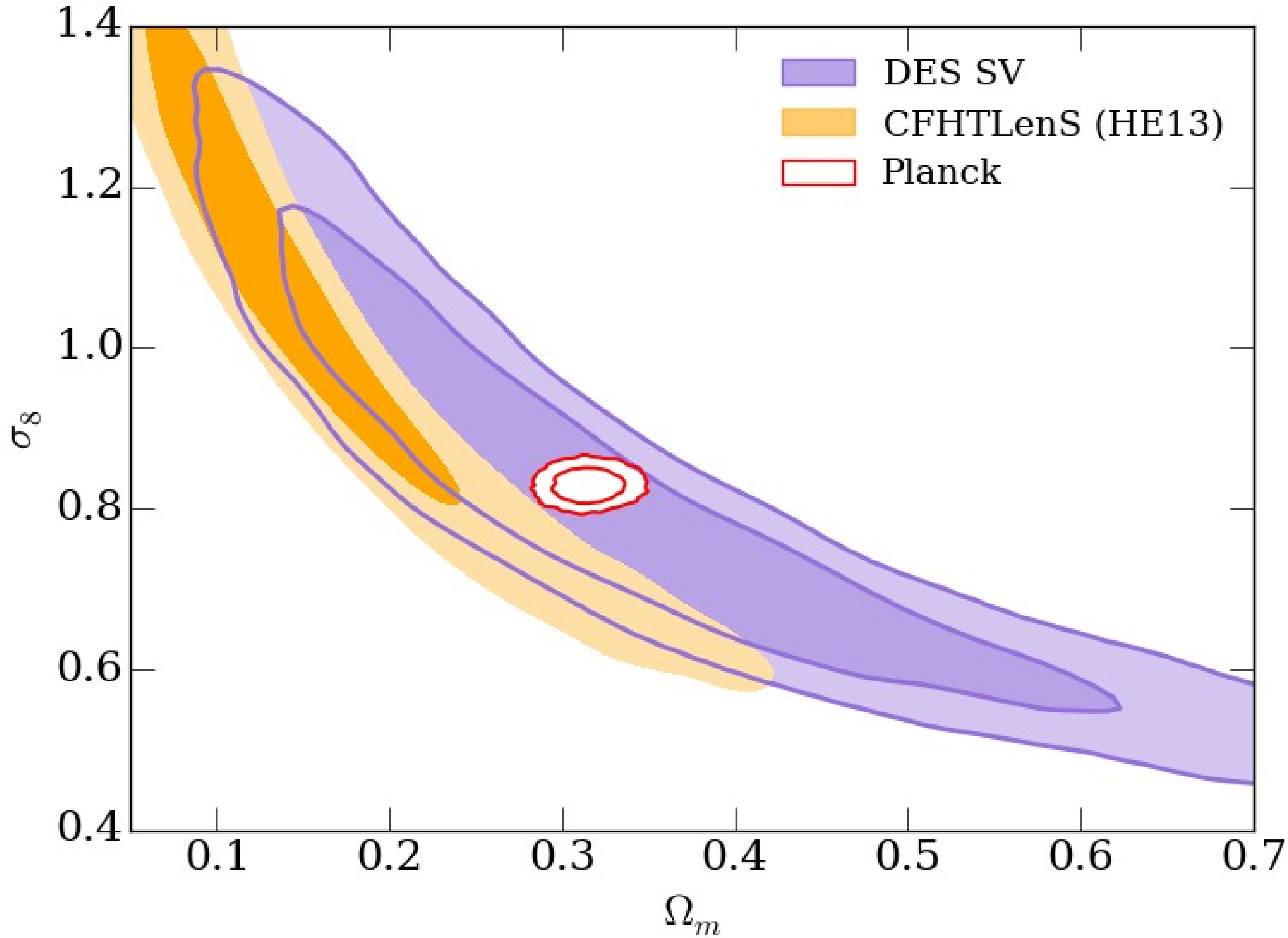


$$\chi^2 = (\vec{x} - \vec{\mu})^T C^{-1} (\vec{x} - \vec{\mu})$$ → minimally possible statistical scatter

$$H^2 = (\vec{x} - \vec{\mu})^T \alpha S^{-1} (\vec{x} - \vec{\mu})$$ → produces too often too good a fit

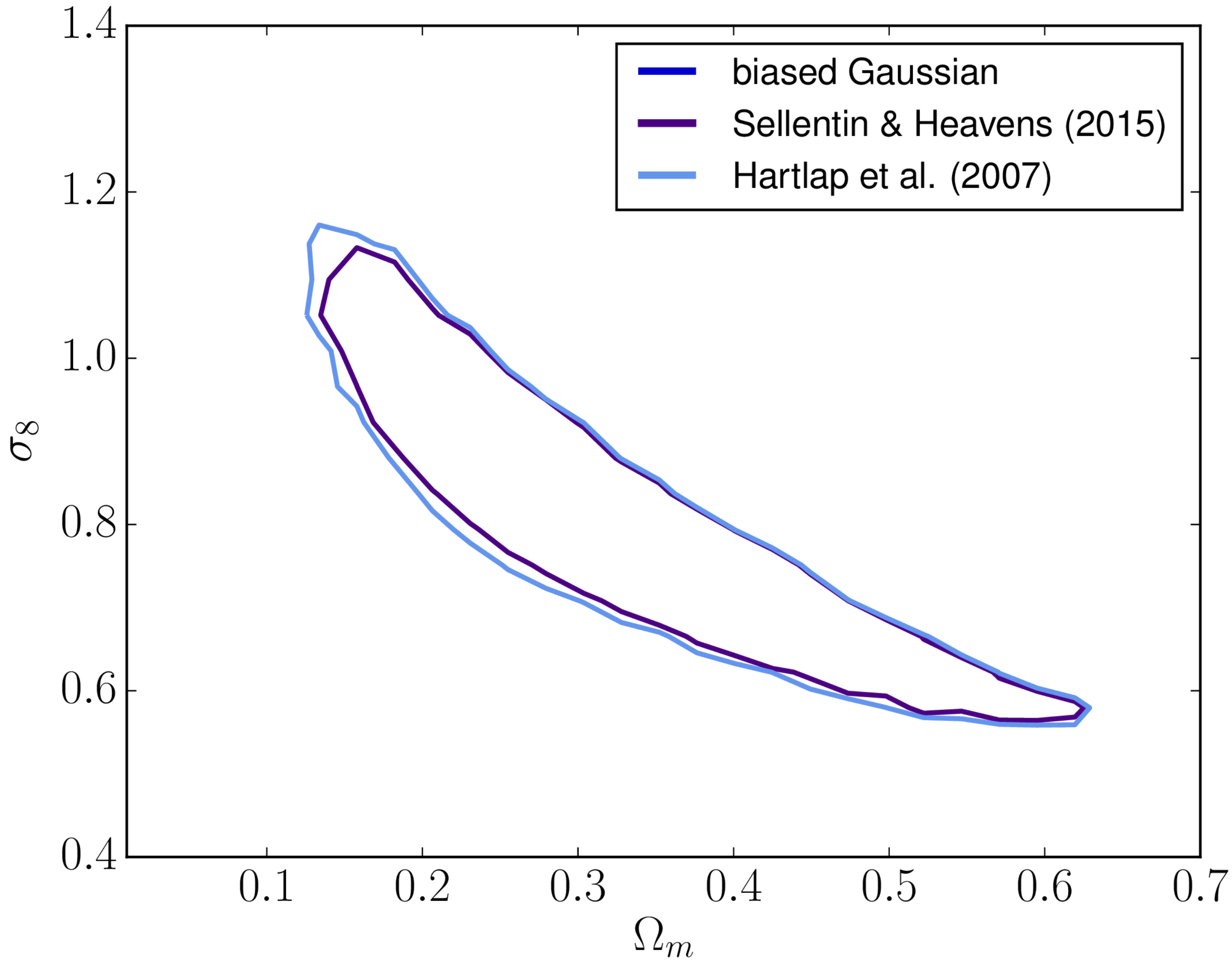
$$T^2 = (\vec{x} - \vec{\mu})^T S^{-1} (\vec{x} - \vec{\mu})$$ → larger scatter due to cov. mat. uncertainty

Dark Energy Survey (SV)

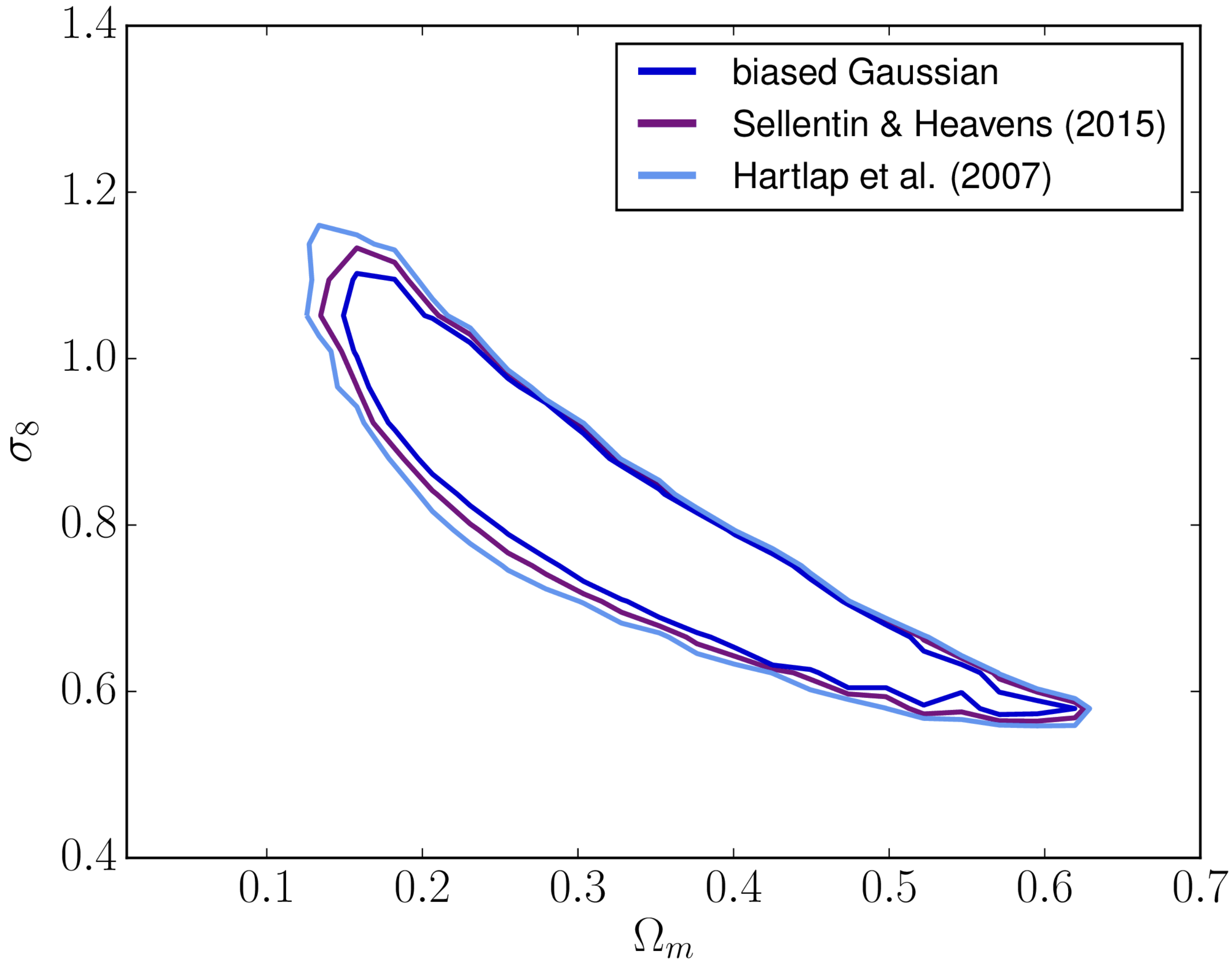


Dark Energy Survey Collaboration (2015), arXiv: 1507:05552

Dark Energy Survey (SV)



Dark Energy Survey (SV)



How many simulations are needed?

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- Aim: beat covariance noise by increasing N (e.g. a Euclid concern)

- Study DES as an example: p = 36, N = 126.
  → compare error bars σ_H (Hartlap et al.'s method)
  σ_{SH} (Sellentin & Heavens)
  σ_o (minimal error for known C)

$$\sigma_H(N=?) \approx \sigma_{SH}(N=126) \quad \rightarrow \quad N=220$$
$$\sigma_H(N=?) \approx \sigma_o \quad \rightarrow \quad N=740$$
$$\sigma_{SH}(N=?) \approx \sigma_o \quad \rightarrow \quad N=420$$

→ Likelihood of Sellentin & Heavens (2015) needs 75% fewer simulations for the same precision

Summary

Updating from Hartlap et al. (2007) to Sellentin & Heavens (2015):

- Changes 3 lines in codes (replace Gaussian likelihood by Eq. (12) from arXiv 1511:05969)
- Keeps the numerical complexity unchanged
- Removes biases in the confidence contours of Hartlap et al.'s method
- Affects the interpretation of goodness of fit & p-values
- For Weak Lensing of DES Science Verification data Sellentin & Heavens (2015) leads to:
  - tighter parameter constraints by about 10%
  - needs 75% fewer simulations
- Reweighting MCMC chains is possible if ln(L) is recorded